The Math Behind the Weather

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Who Am I?

• I was born and raised in New Jersey
• I live in Madison with my wife and three daughters
• I am a climate scientist who has spent that past 2 decades studying how plants, climate, and weather all influence each other
Welcome to CCR

Biogeochemistry

CCR researchers are investigating global and regional biogeochemistry, with a particular focus on the carbon cycle of the land biosphere and oceans and Great Lakes. Using data and models, we seek to elucidate natural carbon fluxes and the processes controlling them, and work to use this knowledge to improve predictive models.

Climate Impacts

Land Surface Processes

Oceanography and Limnology

Past Climates

Who We Are

Since 1948 we have grown into one of the leading departments in our field of Atmospheric and Oceanic Sciences. We have strong graduate and undergraduate programs which are nationally recognized. We graduate about 15 Ph.D. and M.S. students each year; our graduates are active in research labs and universities around the world. We graduate approximately 20 B.S. students each year; they choose options allowing a focus on weather systems or general atmospheric science.

Our faculty of 15 has long maintained breadth and special strength in three areas:

- Climate systems, including the ocean
- Satellite and remote sensing
- Weather systems, including synoptic-dynamic meteorology
News Headlines

- NWS Milwaukee Student Volunteer Opportunity
- Fox River-Southeast Wisconsin Flood Inundation Mapping-New Service
- Seeking CoCoRaHS Volunteers
- Severe Weather Safety/Spotter Training: Thursday, March 15th-Portage 6:30pm

Additional Headlines

Current conditions at Madison, Dane County Regional-Truax Field (KMSN)
Lat: 43.14°N  Lon: 89.35°W  Elev: 860ft.

- Humidity: 88%
- Wind Speed: Calm
- Barometer: 29.87 in (1012.3 mb)
- Dewpoint: 22°F (-6°C)
- Visibility: 7.00 mi
- Last update: 15 Mar 7:53 am CDT

Extended Forecast for Madison WI

<table>
<thead>
<tr>
<th></th>
<th>Today</th>
<th>Tonight</th>
<th>Friday</th>
<th>Friday Night</th>
<th>Saturday</th>
<th>Saturday Night</th>
<th>Sunday</th>
<th>Sunday Night</th>
<th>Monday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weather</td>
<td>Sunny</td>
<td>Clear</td>
<td>Sunny</td>
<td>Slight Chance Snow</td>
<td>Mostly Sunny</td>
<td>Mostly Clear</td>
<td>Sunny</td>
<td>Party Cloudy</td>
<td>Mostly Cloudy then Slight Chance Rain</td>
</tr>
<tr>
<td>High</td>
<td>43 °F</td>
<td>Low: 19 °F</td>
<td>High: 40 °F</td>
<td>Low: 24 °F</td>
<td>High: 46 °F</td>
<td>Low: 27 °F</td>
<td>High: 50 °F</td>
<td>Low: 30 °F</td>
<td>High: 42 °F</td>
</tr>
</tbody>
</table>
Friday and Saturday...Forecast confidence is medium.

A **shortwave trough** will lose **amplitude** while progressing from the central High Plains to the Ohio River valley. Rapid **cyclogenesis** will commence along the Colorado **Front** Range on Friday, with the occluding surface **cyclone** reaching the mid Mississippi and Ohio Valleys by late Saturday. Forecast models are in good agreement with keeping the low, and its associated tight **baroclinic zone**, to our south. However, there is a window Friday night where column **moisture** increases (**Precipitable Water** values ~ 0.5" in southwest Wisconsin), juxtaposed with robust lift and modest **moisture** within the **dendritic** (snow) growth zone. The **GFS ensemble** members are now in better agreement with the surface low track, with most solutions bringing measurable precipitation into southwest Wisconsin on Friday night. This compares favorably with the latest **ECMWF** and Canadian deterministic runs, which suggest the same. The **NAM** appears a rather wet outlier (with > 0.25" of **OPF** southwest of Madison) and is discounted for now. All of this is to say that we’re seeing enough model consistency to increase rain/snow chances on Friday night, particularly for areas southwest of Madison and towards the Illinois border where light snow accumulations are possible.

Sunday through Thursday...Forecast confidence is medium.

**Shortwave** troughing ejects out of the Four Corers region on Sunday, before reaching the lower Missouri and Ohio Valleys by Monday night. The associated surface low passes through the mid Mississippi Valley on Monday with moderate winds to the south.
Advances in Global and Regional Weather Forecasts

Anomaly correlation of ECMWF 500 hPa height forecasts

- Northern hemisphere
- Southern hemisphere

Year


Day 3
Day 5
Day 7
Day 10

%
Climate Model = Weather Model
Major math innovations related to meteorology

• Navier-Stokes equation in turbulent, rotating reference frame
• Chaos in Non-linear dynamical systems
• Numerical solutions and computational approaches to non-linear PDEs
• Analytical geometry
• Statistical Bayesian data assimilation of Earth-atmosphere-ocean observations
“Does the flap of a butterfly’s wings in Brazil set off a tornado in Texas?”
-(Lorenz 1972)
Sensitive dependence to initial conditions

“Finite time for error in representation of small scales to affect accuracy of simulation of large scales, no matter how small in scale and hence amplitude this model error is”

-(Lorenz 1969)

\[
\begin{align*}
\frac{dx}{dt} &= \sigma(y - x) \\
\frac{dy}{dt} &= rx - y - xz \\
\frac{dz}{dt} &= xy - bz
\end{align*}
\]

\[r = 28, \sigma = 10, \text{ and } b = 8/3\]

source: wikipedia
Ensemble Forecast with Initial Uncertainty

Predictable  Semi-predictable  Unpredictable

Ensemble anomaly in X
What's Really Warming the World?
Skeptics of manmade climate change offer various natural causes to explain why the Earth has warmed 1.4 degrees Fahrenheit since 1880. But can these account for the planet’s rising temperature? Watch to see how much different factors, both natural and industrial, contribute to global warming, based on findings from NASA's Goddard Institute for Space Studies.
Transition to Turbulence

Album of Fluid Motion (Van Dyke)
Navier-Stokes
a.k.a Newton’s Second Law for a “Newtonian” Fluid

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \nu \Delta \mathbf{v} + \mathbf{f}(\mathbf{x}, t) \]
N-S Smoothness and Existence Problem is a Millennial Math Problem

1. \( \mathbf{v}(x, t) \in \left[ C^\infty(\mathbb{R}^3 \times [0, \infty)) \right]^3 \), \( p(x, t) \in C^\infty(\mathbb{R}^3 \times [0, \infty)) \)

2. There exists a constant \( E \in (0, \infty) \) such that \( \int_{\mathbb{R}^3} |\mathbf{v}(x, t)|^2 \, dx < E \) for all \( t \geq 0 \).

(A) Existence and smoothness of the Navier–Stokes solutions in \( \mathbb{R}^3 \)

Let \( \mathbf{f}(x, t) \equiv 0 \). For any initial condition \( \mathbf{v}_0(x) \) satisfying the above hypotheses there exist smooth and globally defined solutions to the Navier–Stokes equations, i.e. there is a velocity vector \( \mathbf{v}(x, t) \) and a pressure \( p(x, t) \) satisfying conditions 1 and 2 above.

(B) Breakdown of the Navier–Stokes solutions in \( \mathbb{R}^3 \)

There exists an initial condition \( \mathbf{v}_0(x) \) and an external force \( \mathbf{f}(x, t) \) such that there exists no solutions \( \mathbf{v}(x, t) \) and \( p(x, t) \) satisfying conditions 1 and 2 above.
Going onto a rotating reference frame

\[ \rho \frac{Du}{Dt} = -\nabla \tilde{p} + \mu \nabla^2 u + \frac{1}{3} \mu \nabla (\nabla \cdot u) + \rho g - \rho \left( 2\Omega \times u + \Omega \times \Omega \times x + \frac{dU}{dt} + \frac{d\Omega}{dt} \times x \right). \]
N-S For Earth System

\[
\frac{\partial \rho}{\partial t} = - \frac{\partial \rho u_j}{\partial x_j},
\]

\[
\frac{\partial \theta}{\partial t} = -u_j \frac{\partial \theta}{\partial x_j} + S_\theta,
\]

\[
\frac{\partial u_i}{\partial t} = -u_j \frac{\partial u_i}{\partial x_j} - \frac{1}{\rho} \frac{\partial p}{\partial x_i} - g \delta_{i3} - 2\epsilon_{ijk} \Omega_j u_k,
\]

\[
\frac{\partial q_n}{\partial t} = -u_j \frac{\partial q_n}{\partial x_j} + S_{q_n}, \quad n = 1, 2, 3
\]
One Solution to N-S: Convective instability

- Requires density to increase with height
- Instability occurs when Rayleigh number reaches critical threshold

$$Ra = \frac{h^3 \Delta B}{\nu \kappa} > 1700$$

$$\Delta B = \frac{g \Delta T}{T}$$

Slightly unstable convection in silicone oil

van Dyke p. 82
Cloud Streets: Convective Instability in the Real World
Turbulent Momentum Equation

\[
\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_j \frac{\partial \bar{U}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_i} - g\delta_{i3} + \frac{\mu}{\rho} \frac{\partial^2 \bar{U}_i}{\partial x_j^2}
\]

\[
\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_j \frac{\partial \bar{U}_i}{\partial x_j} + u_j' \frac{\partial u_i'}{\partial x_j'} = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_i} - g\delta_{i3} + \frac{\mu}{\rho} \frac{\partial^2 \bar{U}_i}{\partial x_j^2}
\]

\[
\begin{aligned}
\frac{\partial \bar{U}_i}{\partial t} + \frac{\partial}{\partial x_j} & \left[ \bar{U}_j \bar{U}_i + u_j' u_i' - \nu \frac{\partial \bar{U}_i}{\partial x_j} \right] = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_i} - g\delta_{i3}
\end{aligned}
\]
Numerical Solutions are Necessary

Example of 3-D Grid Box in a Grid Point Model

The COMET Program
SUBROUTINE ADVE(NTSD,DT,DETA1,DETA2,PDTOP &
& ,CURV,F,FAD,F4D,EM_LOC,EMT_LOC,EN,ENT,DX,DY &
& ,HTM,HBM2,VTM,VBM2,LMH,LMV &
& ,T,U,V,PDSL0,TOLD,ULD0,VOLD &
& ,PETDT,UPSTRM &
& ,FWE,FNS,FNE,FSE &
& ,ADT,ADU,ADV &
& ,N_IUP_H,N_IUP_V &
& ,N_IUP_ADH,N_IUP_ADV &
& ,IUP_H,IUP_V,IUP_ADH,IUP_ADV &
& ,IHE,IHN,IVE,IVW,INDEX3_WRK &
& ,IDS,IDE,IDS,JDE,KDS,KDE &
& ,IMS,IME,IMS,JME,KMS,KME &
& ,ITS,ITE,ITS,JTE,KTE)

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$$ SUBPROGRAM DOCUMENTATION BLOCK

! SUBPROGRAM: ADVE HORIZONTAL AND VERTICAL ADECTION

! PRGRMNR: JANJIC ORG: W/NI22 DATE: 93-10-28

! ABSTRACT:

ADVE CALCULATES THE CONTRIBUTION OF THE HORIZONTAL AND VERTICAL
ADVECTION TO THE TENDENCIES OF TEMPERATURE AND WIND AND THEN
UPDATES THOSE VARIABLES.

THE JANJIC ADECTION SCHEME FOR THE ARAKAWA E GRID IS USED
FOR ALL VARIABLES INSIDE THE FIFTH ROW. AN UPSTREAM SCHEME
IS USED ON ALL VARIABLES IN THE THIRD, FOURTH, AND FIFTH
OUTERMOST ROWS. THE ADAMS-BASHFORT TIME SCHEME IS USED.

PROGRAM HISTORY LOG:
87-06-?? JANJIC - ORIGINATOR
95-03-25 BLACK - CONVERSION FROM 1-D TO 2-D IN HORIZONTAL
96-03-28 BLACK - ADDED EXTERNAL EDGE
98-10-30 BLACK - MODIFIED FOR DISTRIBUTED MEMORY
99-07-01 JANJIC - CONVERTED TO ADAMS-BASHFORT SCHEME
COMBINING HORIZONTAL AND VERTICAL ADECTION
02-02-04 BLACK - ADDED VERTICAL CFL CHECK
02-02-05 BLACK - CONVERTED TO WRF FORMAT
02-08-29 MICHALAKES - CONDITIONAL COMPILATION OF MPI
Actual smooth and continuous temperature field in degrees C (similar to spectral model representation)

Grid point model representation of the same temperature field in degrees C
But the Earth is a sphere

What Horizontal Grid? Quasi-uniform for parallel scaling

- skipped lat-lon
- hexagonal-icosahedral
- triangular icosahedral
- kites
- cubed sphere
- Yin-Yang

https://blogs.reading.ac.uk/wcd/2012/02/07/gungho-development-of-a-new-dynamical-core-for-the-unified-model/
Model spread needs to be constrained by data.
Applications of Bayes’ Rule

\[ P(H | e) = \frac{P(e | H) P(H)}{P(e)} \]

Likelihood
How probable is the evidence given that our hypothesis is true?

Prior
How probable was our hypothesis before observing the evidence?

Posterior
How probable is our hypothesis given the observed evidence? (Not directly computable)

Marginal
How probable is the new evidence under all possible hypotheses?
\[ P(e) = \sum P(e | H_i) P(H_i) \]
Ensemble Kalman Filter

**Ensemble Kalman filter** is a Mont Carlo approximation of Kalman Filter. It samples the probability density function (PDF) of **forecast** and **analysis** using ensemble. (Evenson 1994).

\[ X^a = X^f + K(y^o - HX^f) \]

**Before Ensemble Kalman filter**

\[ p^f \approx p^f_e = \langle x^f_i, x^f_i' \rangle \]

**After Ensemble Kalman filter**

\[ p^a \approx p^a_e = \langle x^a_i, x^a_i' \rangle \]

http://slideplayer.com/slide/9781007/

Stochastic filter and Deterministic filter
WPC 120h (5-day) Precipitation Forecast (left) vs. Observed Precipitation (NWS/AHPS, right)

NWS/WPC 120h (5-day) forecast issued 0852 UTC Tue 2/20/18

NWS/AHPS 5-day total observed precipitation ending 12 UTC 2/25/18

Grid max rainfall: 12.96 inches
source: https://www.wmo.int/
THANKS!
Ankur Desai, desai@aos.wisc.edu, @profdesai

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APRIL Group